Magnetic Monopole Content of Hot Instantons

R. C. Brower[a*], D. Chen[b], J. Negele[b], K. Orginos[c] and C-I Tan[d]

[a]Physics Department, Boston University, 590 Commonwealth Ave, Boston, MA 02215, USA

[b]Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

[c]Physics Department, University of Arizona, Tucson, AZ 85721, USA

[d]Physics Department, Brown University, Providence, RI 02912, USA

We study the Abelian projection of an instanton in $R^3 \times S^1$ as a function of temperature (T) and non-trivial holonomic twist (ω) of the Polyakov loop at infinity. These parameters interpolate between the circular monopole loop solution at $T = 0$ and the static 't Hooft-Polyakov monopole/anti-monopole pair at high temperature.

1. INTRODUCTION

Although many qualitative features of QCD are well described by a vacuum state dominated by an instanton "liquid', confinement appears to be an exception [1]. Instead, magnetic monopoles are thought to be the crucial ingredient. This raises the question of how magnetic degrees of freedom can be incorporated into (or reconciled with) an instanton "liquid". A recent step in this direction was taken by Brower, Orginos and Tan (BOT)[2] who studied in detail the magnetic content of a single isolated instanton, defining magnetic currents via the Maximally Abelian (MA) projection. They found a marginally stable direction for the formation of a monopole loop. Now with the more general caloron solution of T. Kraan and P. van Baal [3], and K. Lee and C. Lu [4], this analysis can be extended to an isolated SU(2) instanton at finite temperature (T) with a non-trivial holonomy (ω) for the Polyakov loop.

The resultant picture that emerges is appealing (see Fig. 1). For $\omega = 0$, the small monopole loop at the core of a cold instanton grows in size as one increases the temperature and is transformed into a single static 't Hooft-Polyakov monopole at infinite temperature, as noted earlier by Rossi [5]. Note that the other quadrants of Fig. 1 can be found by applying Z_2 center symmetry ($\omega \to \omega + \frac{1}{2}$) and monopole to anti-monopole charge conjugation ($\omega - \frac{1}{4} \to \frac{1}{4} - \omega$).

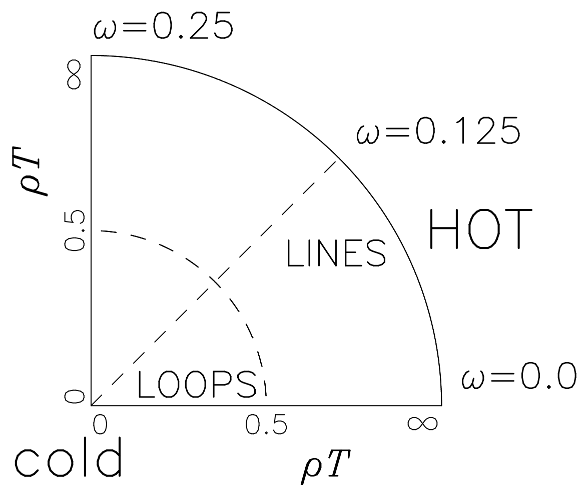

Figure 1. Magnetic monopole phase plane for the caloron in polar co-ordinates: $(\rho T, 2\pi\omega)$.

The MA projection provides a fully gauge and Lorentz invariant definition of monopole currents by introducing an auxiliary adjoint Higgs field, $\vec{\phi}(x)$, fixed at the classical minimum,

$$G = \frac{1}{2}\int [(D_\mu(A)\vec{\phi})^2 + \lambda(\vec{\phi}\cdot\vec{\phi} - 1)^2] d^4x \;, \qquad (1)$$

in a fixed background gauge field, A_μ. This yields the Abelian projected field strength,

$$f_{\mu\nu} = \mathbf{n}\cdot\mathbf{F}_{\mu\nu} - \mathbf{n}\cdot D_\mu\mathbf{n}\times D_\nu\mathbf{n} \;, \qquad (2)$$

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with its U(1) monopole current,

$$k_\mu = \frac{1}{4\pi}\partial_\nu \tilde{f}_{\nu\mu} \to \frac{1}{8\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \mathbf{n}\cdot\partial_\rho \mathbf{n}\times\partial_\sigma \mathbf{n}\ , \quad (3)$$

where $\mathbf{n}(x) \equiv \vec{\phi}(x)/|\phi(x)|$. It is conventional to identify the MA gauge by the rotation $\Omega(x)$ in the coset $SU(2)/U(1)$ that aligns $\mathbf{n}$ along the 3 axis. We have extended the conventional Abelian projection ($\lambda = \infty$) to a continuous family including the analytically more tractable BPS limit ($\lambda = 0$), where the difficult problem of minimizing the MA functional G reduces to an eigenvector problem for the Higgs field, $D_\mu(A)^2\vec{\phi}_E = E\vec{\phi}_E$.

2. COLD MONOPOLE LOOP

We begin at the origin of Fig. 1, where there is a single isolated instanton at zero temperature. A trivial, but essential, observation is that the singular gauge instanton in the 't Hooft ansatz,

$$A^a_\mu = \bar{\eta}^a_{\mu\nu}\partial_\nu \log(1+\rho^2/x^2). \quad (4)$$

is **also** the MA projection which minimizes the Higgs action G. In the BPS limit, this is equivalent to having a zero eigenvalue solution,

$$\vec{\phi}_0(x) = \frac{x^2}{x^2+\rho^2}[0,0,1],$$

for the Higgs field aligned with the 3-axis. Consequently there is **no** magnetic content to the MA projection. This would be the entire story except that there is another zero eigenvalue that implies a flat direction for the formation of an infinitesimal monopole loop.

The geometry of this loop is interesting. The gauge singularity at the origin is caused by a rotation, $g(x) = x_\mu\tau_\mu/|x|$ and $\tau_\mu = (1, i\vec{\tau})$. Locally it is advantageous to "unwind" this singularity to a distance R further reducing the MA functional G. This almost wins, creating a monopole loop of radius R which only slightly increases G, $\delta G/G \simeq R^4\log(R)$. Almost any local disturbance, due to a nearby instanton for example, will stabilize the loop [2]. The second zero mode vector in the BPS limit is easily constructed using conformal invariance.

$$\vec{\phi}_1(x) = \frac{1}{x^2(x^2+\rho^2)}[\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta],$$

where $\alpha = \varphi - \psi$, $\cos\beta = (v^2-u^2)/(u^2+v^2)$, with $x_1 + ix_2 = ue^{i\varphi}$, $x_3 + ix_0 = ve^{\psi}$.

The superposition of the two zero modes produces a loop. Finally note that the topological charge is related to the magnetic charge, through a surface term on the boundary of the loop, Σ,

$$Q \to \int_\Sigma \frac{Tr[\Omega_R d\Omega_R^\dagger \wedge \Omega_R d\Omega_R^\dagger \wedge \Omega_R d\Omega_R^\dagger]}{24\pi^2}\ ,$$

where Ω_R is the singular gauge transformation providing a Hopf fibration for the infinitesimal "non-contractible" loop.

3. HOT BPS MONOPOLES

At low temperature with $\omega = 0$ the MA projection is similar. The periodic instanton in the 't Hooft ansatz,

$$A^a_\mu = \bar{\eta}^a_{\mu\nu}\partial_\nu \log\left(1+\frac{\pi T\rho^2\sinh(2\pi Tr)}{2r(\sinh^2(\pi Tr)+\sin^2(\pi Tt))}\right),$$

is again equivalent to the MA projection. "Unwinding" the periodic copies of the singularities at $x = 0$ is now accomplished by $g = X_\mu\tau_\mu/|X|$, $X_\mu = [\tan(\pi Tt), \hat{r}\tanh(\pi Tr)]$, leaving a monopole loop. However surprisingly at infinite temperature, or equivalently $\rho = \infty$ as noted by Rossi, the instanton is gauge equivalent to the static 't Hooft-Polyakov monopole solution. With $\mathbf{n}(x) = \hat{r}$, this is the correct MA projection (or unitary gauge). For the case of the BPS limit, the solution is simply $\vec{\phi}(x) \equiv \vec{A}_0(x)$, as one might expect. Consequently the MA projection correctly identifies the standard static monopole.

4. CALORON T-ω PLANE

For $\omega \neq 0$, we encounter the full complexity of the new caloron solution [3],

$$A_\mu = \tau^3\bar{\eta}^3_{\mu\nu}\partial_\nu\log\phi(x) + (\tau^+\bar{\eta}^-_{\mu\nu}\partial_\nu\chi(x) + c.c.)\psi(x),$$

in the singular gauge. In the limit of $T \to \infty$, we have verified that MA projection now gives a pair of 't Hooft-Polyakov BPS monopole/antimonopole separated by distance $D = \pi\rho^2 T$ as expected. However, now the singular gauge caloron no longer satisfies the MA projection and it is difficult to find the MA projection analytically.

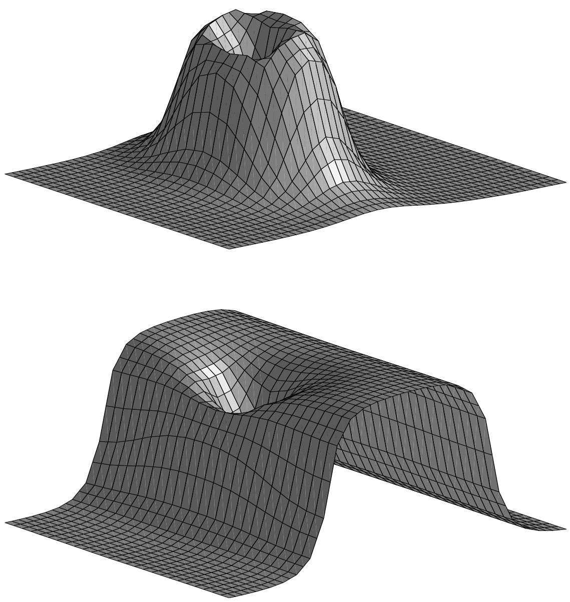

Figure 2. Profiles of $\beta(0,z,t)$ at $\omega = .125$ for a loop (top, $\rho T = 0.56$) and for monopole/anti-monopole pair (bottom, $\rho T = 0.57$).

Thus we have minimized G numerical in the interior of the $T-\omega$ phase plane of Fig. 1 by placing the functional on a grid. On symmetry grounds, one can prove the existence of cylindrical solutions with $\alpha = \varphi + \tilde{\alpha}(u,z,t)$, $\beta(u,z,t)$. The Dirac sheet is located by a jump in $\beta(t,x)$ by π. In Fig. 2, we give the profile for $\beta(u,z,t)$ in the z-t plane slicing through the instanton centered at $u = 0$.

To explore further the transition from the monopole loop to a pair of monopole lines, we plot in Fig. 3 the area of the minimal spanning Dirac sheet. At $\rho T \simeq 0.56$, there is a clear transition separating the two regimes.

Based on the absence of a loop for a single isolated instanton [2], we anticipate that the formula for the size of the loop (or separation of the lines) must involve a new length scale, L. This scale represents the distance to nearby perturbations such as the anti-instanton presented in Ref. [2]. For the single caloron plotted here, the new scale is $L = \beta = 1/T$. This suggests a simple scaling form: $R \sim \rho(\rho T)^\gamma$ with $\gamma > 0$. Indeed in Fig. 3 for $(\rho T)^2 < 0.32$, we do see a positive curvature for the area, $A/\beta^2 \sim (RT)^2 \sim (\rho T)^{2+2\gamma}$, *i.e.*, $\gamma > 0$, consistent with our expectation. On the other hand, at high temperatures, the monopole/anti-monopole trajectories are known [3,4] to be separated asymptotically by $D = \pi\rho(\rho T)^\gamma$ with $\gamma = 1$, which is also confirmed by a linear fit to $A = D\beta \simeq \pi\rho^2$ for $(\rho T)^2 > 0.3$.

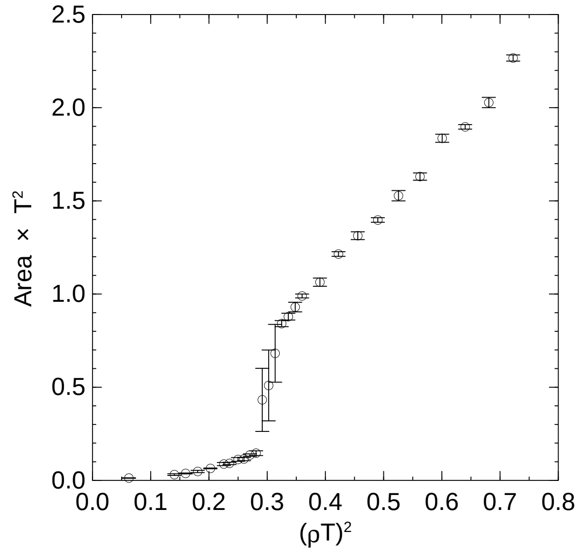

Figure 3. The area A of the minimal spanning Dirac sheet versus temperature T for $\omega = 0.125$.

Finally it is interesting to note that the kinematical "transition" seen in Fig. 3 is near to the Yang-Mills deconfinement temperature for a typical instanton size of $\frac{1}{3}$ fermi. However, a serious analysis of deconfinement dynamics and its possible relations to the monopole content of the caloron is left to future investigations.

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